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A CREEP BENDING ANALYSIS OF PLATES BY THE FINITE ELEMENT METHOD

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Abstract—The finite element method of structural analysis is applied to the nonlinear problem of stationary creep bending of thin plates. The finite element formulation is derived from a variational theorem analogous to the theorem of stationary potential energy and results in a set of simultaneous nonlinear algebraic equations which are solved using the Newton–Raphson method.

Two fully conforming refined plate elements, a six degree of freedom annular element and an eighteen degree of freedom triangular element are adapted for use in the solution of stationary creep problems. Examples considered are simply supported circular plates under a uniform load and a point load and a uniformly loaded simply supported square plate.

1. INTRODUCTION

THE simplifications resulting from the assumptions of small deformations and stationary creep, that is creep under a constant state of stress, have allowed a number of closed form and numerical solutions to a variety of creep problems. Wahl has considered for example the creep of rotating discs using several creep laws modelled on the theory of plasticity [1–3]. Similarly Venkatraman and Hodge [4, 5], using a maximum shear stress flow law have obtained solutions for the stationary creep of uniformly loaded simply supported and clamped circular plates. These plate problems have been solved as well by Odqvist [6] and by Bentson *et al.* [7] using a variational method. Solutions for compressible circular and annular plates have been obtained by Patel *et al.* [8, 9] and solutions for cylinders and spheres subjected to internal pressures are easily obtained [10, 11].

It is apparent that almost all the problems for which solutions have been obtained exhibit a very high degree of symmetry and even with this simplification it is in general necessary to resort to either numerical methods to solve the governing differential equations directly or to use classical variational methods. The application of the finite element method, which is particularly suited to problems with complex geometries and loadings, is therefore a natural extension.

This paper outlines a procedure [12] in which the problem of stationary creep of thin plates is cast in a finite element form using the creep analogue of the theorem of stationary potential energy. The resulting nonlinear algebraic equations are solved using the Newton–Raphson method. Two high precision plate elements are used, a six degree of freedom annular element for axisymmetric problems and an eighteen degree of freedom triangular element. Several example problems are solved.

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2. CREEP MODEL

Although the method described here is not restricted to the use of any particular creep law, the remainder of the development is based on the following creep law given by Odqvist for an incompressible isotropic material.

$$e_{ij} = \frac{3\sigma_n}{2\varepsilon_n}\left(\frac{S}{\sigma_n}\right)^{n-1} s_{ij}. \tag{1}$$

This is a three dimensional generalization of the Norton power law model for uniaxial creep. The components of the rate of deformation tensor e_{ij} referred to a Cartesian co-ordinate system are given in terms of the velocity v_i by

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right).$$

The components of the stress deviator tensor in terms of the stress tensor are

$$s_{ij} = \sigma_{ij}-\tfrac{1}{3}(\sigma_{kk})\delta_{ij}$$

with the invariant S defined by

$$S = \sqrt{(\tfrac{3}{2}s_{ij}s_{ij})}.$$

The quantities σ_n and n are material constants and ε_n is an arbitrary standard constant equal to the strain rate of a uniaxial specimen under a stress σ_n.

For a material having the constitutive equation (1), the work function

$$U(e_{ij}) = \int s_{ij}\,\mathrm{d}e_{ij}$$

described by Hill [13] is given by

$$U = \frac{n}{n+1}\varepsilon_n\sigma_n\left(\frac{e}{\varepsilon_n}\right)^{(n+1)/n}$$

where the invariant e is defined as

$$e = \sqrt{(\tfrac{2}{3}e_{ij}e_{ij})}.$$

Furthermore for quasi-static creep of a body of volume V with prescribed surface tractions T_i on part S_T of the surface, prescribed velocities $v_i = V_i$ on part S_V of the surface and a body force F_i/unit volume, the functional

$$\pi = \int_V (U(e_{ij})-F_iv_i)\,\mathrm{d}V - \int_{S_T} T_iv_i\,\mathrm{d}S_T$$

evaluated over the set of all velocity fields satisfying the velocity boundary conditions on S_V has a stationary value for the actual velocity field in the body [6]. This theorem which is analogous to the theorem of stationary potential energy in the theory of elasticity is used here as a basis for the finite element creep model.

Consider now an initially flat plate of thickness $2h$, whose middle surface coincides with the xy plane of a Cartesian coordinate system. Denoting by w the velocity of the middle surface in the z direction, the usual assumptions of small deflection thin plate theory and the assumption of material incompressibility give

$$e^2 = z^2\kappa^2$$

where†

$$\kappa^2 = \tfrac{4}{3}(w_{xx}^2 + w_{yy}^2 + w_{xx}w_{yy} + w_{xy}^2) \tag{2}$$

is called the curvature rate invariant. For a plate acted upon by a distributed load $q_0(x, y)$ in the z direction the functional π takes the form

$$\pi = \int_A U(\kappa)\,\mathrm{d}A - \int_A q_0 w\,\mathrm{d}A$$

where

$$U(\kappa) = \frac{n}{n+1}\kappa_n M_n\left(\frac{\kappa}{\kappa_n}\right)^{(1+n)/n},$$

$$\kappa_n = \frac{\varepsilon_n}{h},$$

and

$$M_n = \frac{2n\sigma_n h^2}{2n+1}.$$

The moment–curvature rate relations associated with the constitutive equation (1) are

$$M_x = -\frac{4}{3}\frac{M_n}{\kappa_n}\left(\frac{\kappa}{\kappa_n}\right)^{(1-n)/n}(w_{xx} + \tfrac{1}{2}w_{yy})$$

$$M_y = -\frac{4}{3}\frac{M_n}{\kappa_n}\left(\frac{\kappa}{\kappa_n}\right)^{(1-n)/n}(w_{yy} + \tfrac{1}{2}w_{xx})$$

and

$$M_{xy} = -\frac{2}{3}\frac{M_n}{\kappa_n}\left(\frac{\kappa}{\kappa_n}\right)^{(1-n)/n}w_{xy}.$$

3. FINITE ELEMENT FORMULATION

In this section the equilibrium equations for an arbitrary single plate element referred to a local coordinate system are derived. The equilibrium equations for each element may then be transformed to a common global coordinate system and assembled in the usual manner [14] to give the equilibrium equations for the complete structure. The kinematic boundary conditions are then applied and the set of algebraic equations solved for the nodal velocity vector.

† The subscripts x and y on w, and on ϕ later in the text denote partial differentiation.

Consider a typical element referred to a local Cartesian coordinate system. The distribution of the velocity in the z direction is approximated by

$$w = \{\phi(x, y)\}^T\{a\} \tag{3}$$

where the vector $\{a\}$ is a vector of generalized velocities and the elements of the vector function $\{\phi\}$ are polynomials in x and y. The nodal velocity vector $\{w\}$ has as elements the velocities and spatial derivatives of velocities at the element nodal points. The vectors $\{w\}$ and $\{a\}$ are in general related by

$$\left\{\begin{array}{c} w \\ \hline 0 \\ \vdots \\ 0 \end{array}\right\} = [T]\{a\} \tag{4}$$

where depending on the particular type of element considered it may be necessary to add a number of constraint equations on the elements of $\{a\}$, as shown, in order to enforce continuity of velocities and the normal derivative of velocity across element boundaries. Inverting equation (4) gives

$$\{a\} = [T_1]\{w\}$$

with $[T_1]$ consisting of the first m columns of $[T]^{-1}$ where m is the dimension of $\{w\}$.

With this approximation for the velocity distribution within a typical element, the curvature rate invariant may be expressed as

$$\kappa^2 = \{a\}^T[B]\{a\}$$

where the symmetric matrix $[B]$ is equal to

$$[B(x, y)] = \tfrac{4}{3}[\{\phi_{xx}\}\{\phi_{xx}\}^T + \{\phi_{yy}\}\{\phi_{yy}\}^T + \tfrac{1}{2}(\{\phi_{xx}\}\{\phi_{yy}\}^T + \{\phi_{yy}\}\{\phi_{xx}\}^T) + \{\phi_{xy}\}\{\phi_{xy}\}^T].$$

The domain of the functional π for a typical element is considered to be the set of all velocity fields of the form (3) and which also satisfy the kinematic boundary conditions. The first variation of the functional π for an element loaded by a distributed load $q_0(x, y)$, is then equal to

$$\delta\pi = \delta\{w\}^T[k]\{w\} - \delta\{w\}^T\{p\}$$

where

$$[k] = \frac{M_n}{\kappa_n}[T_1]^T \int_A \left(\frac{\kappa}{\kappa_n}\right)^{(1-n)/n} [B]\, dA[T_1] \tag{5}$$

is called the creep stiffness matrix and

$$\{p\} = [T_1]^T \int_A \{\phi\} q_0\, dA$$

is the consistent load vector for a distributed load on the element.

From the variational principle stated in Section 2 the element equilibrium equations are obtained. Assembling these in the usual manner gives the equilibrium equations

$$[K]\{W\} = \{P\} \tag{6}$$

for the complete plate.

The stiffness matrix in equation (5) differs from that for the analogous incompressible linear elastic plate element by the factor $(\kappa/\kappa_n)^{(1-n)/n}$ in the integrand. That is, for the creep problem, the so called creep stiffness matrix is itself a function of velocity. The equilibrium equation (6) is therefore nonlinear and cannot be solved by a simple matrix inversion.

The Newton–Raphson method was used in this analysis to solve the nonlinear equilibrium equations. This procedure is now briefly outlined.

Consider the vector function

$$\{F\} = [K]\{W\} - \{P\}.$$

Expanding $\{F\}$ in a Taylor series about an approximate solution $\{W_0\}$ gives

$$\{F(W_0+\Delta W)\} = \{F(W_0)\} + \left[\frac{\partial\{F\}}{\partial\{W\}}\right]_{W=W_0}\{\Delta W\} + \text{higher order terms in } \{\Delta W\}. \tag{7}$$

It follows from equation (5) that the gradient matrix is given by

$$\frac{\partial\{F\}}{\partial\{W\}} = [K]+[G]$$

where the matrix $[G]$ is obtained by assembly of the matrix

$$[g] = \left(\frac{1-n}{n}\right)\frac{M_n}{\kappa_n}[T_1]^T\int_A\left(\frac{\kappa}{\kappa_n}\right)^{(1-3n)/n}[B]\{a\}\{a\}^T[B]\,\mathrm{d}A[T_1]$$

for each element in the same manner that the assembled stiffness matrix $[K]$ is formed. Neglecting the higher order terms in equation (7) gives the Newton–Raphson iteration formula at the $(m+1)$ st iteration.

$$\{W\}_{m+1} = \{W\}_m + ([K]+[G])_m^{-1}(\{P\}-[K]_m\{W\}_m).$$

The solution procedure adopted was as follows. For a given plate geometry and loading the linear problem was solved, that is with n equal to 1. The velocity solution obtained was then scaled by a factor ψ, the result being used as the starting point in the search for the solution to the same problem but with n equal to two. Proceeding in this manner the solutions corresponding to an ascending sequence of values of the creep exponent n were obtained. The factor ψ used was determined for a particular problem on the basis of prior estimates for the maximum velocity in the plate and was chosen such that the maximum velocity in the distribution used as a starting point, was of the same order of magnitude as the estimated maximum velocity.

4. ANNULAR PLATE ELEMENT

A six degree of freedom high precision fully-conforming annular plate element was derived for the solution of axisymmetric creep bending problems. The element nodal velocity vector is

$$\{w\}^T = \{w_1, w_{r_1}, w_{rr_1}, w_2, w_{r_2}, w_{rr_2}\}$$

where the subscripts 1 and 2 denote the inner and outer edges of the element, respectively. The velocity function associated with the element is taken to be a complete fifth order polynomial in the radius r. The six polynomial coefficients match the number of nodal degrees of freedom and the derivation of the transformation matrix $[T_1]$ and the matrices $[k]$ and $[g]$ proceeds as described in Section 2. A more detailed description is given in Ref. [12].

To obtain a circular element the inner radius is set to zero and the condition of zero slope at the plate centre applied.

The integrations over the element area, which are required in the calculation of the matrices $[k]$ and $[g]$ must be performed numerically. The results presented in this paper were obtained using the four point Gaussian Quadrature Formula.

5. TRIANGULAR PLATE ELEMENT

A triangular plate element was used to obtain stationary creep bending solutions for plates with more complex geometries. The element chosen was the eighteen degree of freedom fully conforming plate element developed by Cowper *et al.* [15, 16].

The element is referred to a local Cartesian coordinate system with the x axis coinciding with one edge of the element and the y axis passing through the third vertex. The degrees of freedom at each of the vertices are w, w_x, w_y, w_{xx}, w_{xy} and w_{yy}. The velocity distribution within the element is taken as a restricted quintic polynomial.

$$w = a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^3 + a_8\xi^2\eta + a_9\xi\eta^2 + a_{10}\eta^3 + a_{11}\xi^4$$
$$+ a_{12}\xi^3\eta + a_{13}\xi^2\eta^2 + a_{14}\xi\eta^3 + a_{15}\eta^4 + a_{16}\xi^5 + a_{17}\xi^3\eta^2 + a_{18}\xi^2\eta^3 + a_{19}\xi\eta^4 + a_{20}\eta^5.$$

Two constraint equations result from conformity requirements on the normal derivatives of velocity along the element edges thus reducing the number of independent polynomial coefficients to eighteen.

The formulation of the stiffness matrix and the matrix $[g]$ for this element proceeds as described in Section 2. A more detailed discussion is found in Ref. [12].

The numerical integrations required during the calculation of $[k]$ and $[g]$ were performed using a 13 point Gaussian type integration formula for triangular regions. This formula which was derived by Cowper [17], is exact for polynomials of seventh order in ξ and η.

6. DISCUSSION

The finite element method described here for the creep bending analysis of thin plates was used in the solution of several example problems. Among the problems solved were a simply supported circular plate under a uniform load and a point load, and a simply supported uniformly loaded square plate.

The finite element solution for the uniformly loaded circular plate problem was obtained using the annular element described in Section 4. Four elements were used with the element boundaries at r/a equal to 0·25, 0·5, 0·75 and 1·0. A summary of the maximum values for velocity and bending moment is given in Table 1 for values of n from 1 to 9.

TABLE 1. UNIFORMLY LOADED SIMPLY SUPPORTED CIRCULAR PLATE

n	At centre $\frac{w}{\kappa_n a^2}\left(\frac{6M_n}{q_0 a^2}\right)^n$	At centre $\frac{6M_r}{q_0 a^2}$
1	0·25781	1·31250
3	0·23618	1·12936
5	0·20552	1·06117
7	0·17650	1·02628
9	0·15076	1·00533

Patel *et al.* [9] have used an iterative procedure to solve the differential equations for this problem with an equivalent creep law. With n equal to 3, 7 and 15 (their $\nu = \frac{1}{2}$, $m = 1, 3, 7$) the finite element solutions for both velocity and bending moment were found to agree to four figures with the tabulated solutions of Ref. [9].

Figures 1 and 2 give the results for the point loaded circular plate. A much finer gridwork was used because of the large curvatures in the vicinity of the point load. The element boundaries are indicated in Fig. 1.

The only other solution which could be found for this problem is that obtained by Malinin and reproduced by Kachanov [18]. Malinin used the Ritz method with an assumed

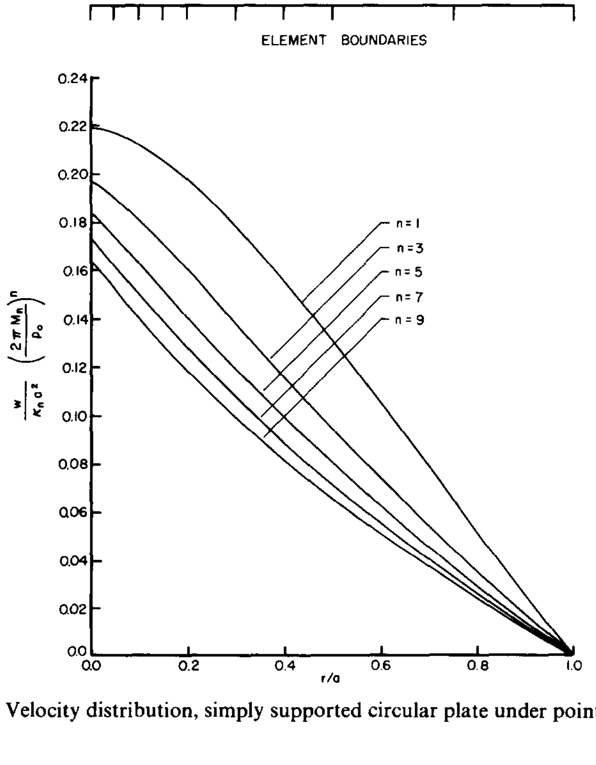

FIG. 1. Velocity distribution, simply supported circular plate under point load.

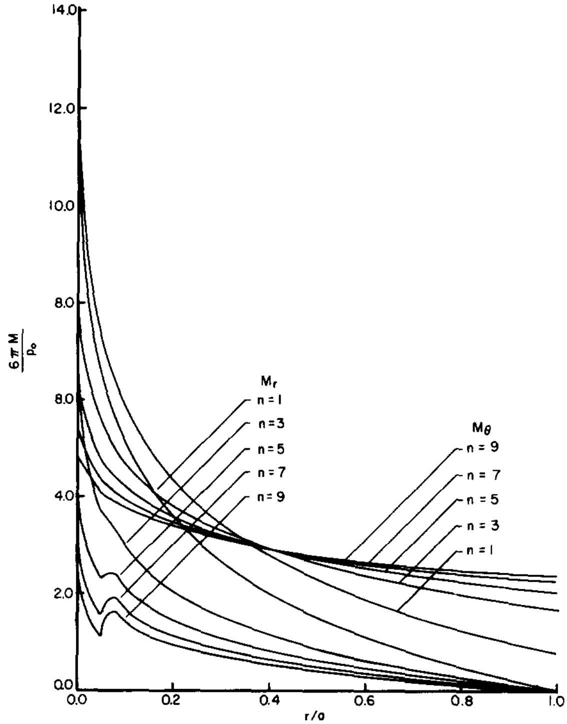

FIG. 2. Bending moment distribution, simply supported circular plate under point load.

velocity distribution of the form

$$w = c_1 w_1$$

where w_1 is the displacement solution for the corresponding incompressible Hookean elastic plate. Thus the solution obtained is exact only for $n = 1$ and provides only a rough approximation to the velocity solution as n becomes larger than one. For example with $n = 3$ the finite element solution obtained here gives the velocity at the centre as

$$w = 0{\cdot}197\kappa_n a^2 \left(\frac{p_0}{2\pi M_n}\right)^3$$

whereas Malinin indicates a value of the order of

$$0{\cdot}15\kappa_n a^2 \left(\frac{p_0}{2\pi M_n}\right)^3.$$

The bending moment curves obtained for this problem are found to exhibit cusps at the element nodal lines since the annular element used ensures continuity of the second order derivatives of velocity at nodal lines while higher order derivatives may be discontinuous. The cusps are most apparent in the vicinity of the point load and become more severe as n increases.

The uniformly loaded simply supported square plate problem is presented here as an example of a creep problem which by direct methods is intractable. Classical variational methods may be used, which according to Rabotnov [19] has been done by Kachanov for rectangular plates. A solution using Reissner's variational principle has been obtained by Ranlet [20].

Four gridworks, shown in Fig. 3, were used in the finite element analysis. Convergence in velocity was obtained with the 16 element representation of one eighth of the plate, with the convergence in bending moments being somewhat slower, as expected. The maximum values obtained for the velocities, bending moments, and twisting moment are given in Table 2, for values of n from 1 to 9. The distributions of these quantities are given in Figs. 4–6, along with the solutions obtained in Ref. [20] for n equal to 1 and 3.

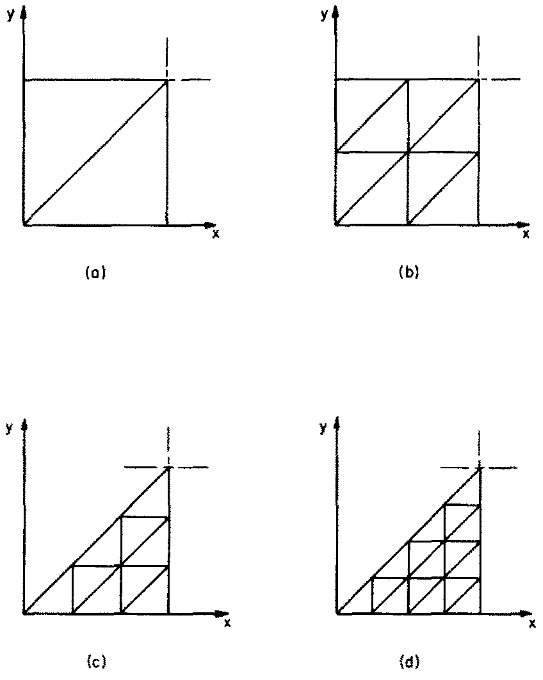

FIG. 3. Element gridworks for square plate.

TABLE 2. UNIFORMLY LOADED SIMPLY SUPPORTED SQUARE PLATE

n	At plate centre $\frac{w}{\kappa_n L^2}\left(\frac{20M_n}{q_0 L^2}\right)^n$	At plate centre $\frac{3M_x}{4q_0 L^2}$	At corner $\frac{3M_{xy}}{4q_0 L^2}$
1	0·060935	0·041438	−0·0174713
3	0·039845	0·035829	−0·0172597
5	0·025379	0·033840	−0·0172847
7	0·016108	0·032850	−0·0172980
9	0·010224	0·032262	−0·0173044

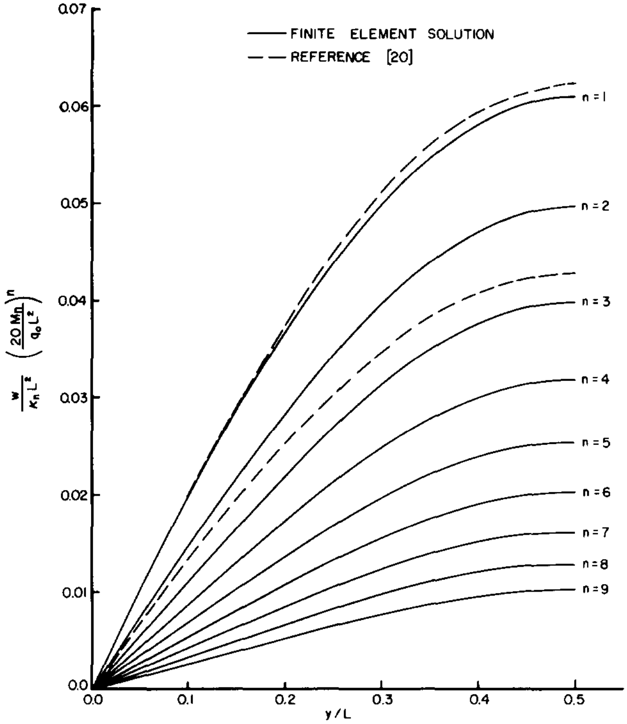

FIG. 4. Velocity along $x/L = 0{\cdot}5$, simply supported square plate under uniform load.

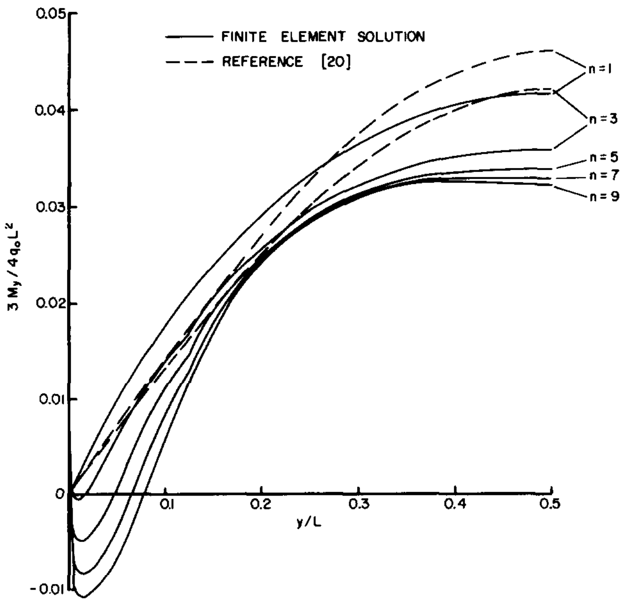

FIG. 5. Bending moment along $x/L = 0{\cdot}5$, simply supported square plate under uniform load.

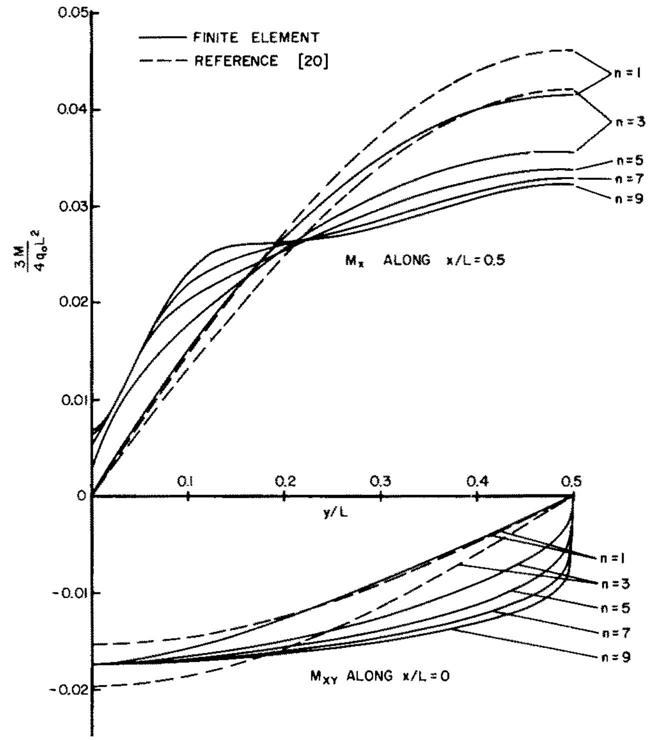

FIG. 6. Bending moment and twisting moment distribution, simply supported square plate under uniform load.

It is observed that there is rough agreement between the results of Ranlet and the finite element solution. For example with n equal to 3 the difference in velocity solutions at the centre is about 8 per cent and in the maximum bending moment, 17 per cent. The distributions for bending moments and twisting moment differ significantly. However the results of Ref. [20] were obtained using very simple shape functions for velocity and bending moments and even for the linear problem, $n = 1$, the solution differs from the exact solution by 4 per cent in velocity and 12 per cent in bending moment at the plate centre. In contrast the maximum velocity obtained by the finite element method agrees to six figures with the exact solution and the maximum bending moment differs by less than 0·01 per cent. It may be argued therefore that for n other than 1 the finite element solutions are likely to be in closer agreement to the real solution.

The finite element solution for the bending moments exhibits some cusping, becoming more severe as n increases. In addition the bending moments along the centreline are found not to be zero at the plate edge, for $n > 1$. Plots of bending moment vs. y along any other line $x = 0$ were found to be well behaved with both bending moments remaining positive and becoming zero at the edge. The problem along the centreline results from two factors; the form of the moment–curvature rate relations and the fact that since the finite element model is derived from a kinematic variational principle, only kinematic boundary conditions are satisfied explicitly. Thus the boundary condition $w_{yy} = 0$ along $y = 0$, will be satisfied only in the limit as the element gridwork is refined. For the 16 element gridwork, with $n = 9$, the maximum value of w_{yy} along the edge is less than 1 per cent of the value at the plate centre so that this condition is essentially satisfied. Furthermore it is found that along the centreline, for the larger values of n, curves of w_{xx} and w_{yy} vs. y, tend to become tangent to the y axis near the origin.

Consider now the expression for the bending moment M_y which along the centreline becomes

$$M_y = -\tfrac{4}{3}M_n(\kappa_n)^{1/n}\{\tfrac{4}{3}(w_{xx}^2 + w_{yy}^2 + w_{xx}w_{yy})\}^{(1-n)/2n}\{w_{yy} + \tfrac{1}{2}w_{xx}\} \tag{8}$$

since w_{xy} is zero. As y becomes zero, w_{xx} and w_{yy} which are of the same order of magnitude both approach zero. Thus the last two terms in equation (8) behave like $w_{yy}^{1/n}$ as $w_{yy} \to 0$. Clearly a small absolute error in w_{yy} can result in a much larger error in the bending moment, the error increasing as n increases. This is a very localized disturbance however and away from this region the relative errors in the curvature rates become much smaller resulting in an even greater decrease in the relative error in the bending moments for $n > 1$. This characteristic of the finite element bending moment results could be eliminated by use of a model based on the complementary variational principle for stationary creep [6], however for problems where creep deflections and creep velocities are of primary interest the kinematic model is preferred.

Additional errors in the finite element solution arise from the use of numerical integration in calculating the creep stiffness matrices. However the excellent agreement found between the results of Ref. [9] for the uniformly loaded simply supported circular plate and the finite element solution obtained here using only four elements with a low order integration formula indicates that the use of higher order integration formulae, with a resulting increase in computation time is not warranted.

Although the presentation here is limited to thin plate bending, the method is equally applicable to three dimensional problems. The method provides a versatile and easily applied approach to obtaining numerical solutions to stationary creep problems with complex geometries and loadings.

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Абстракт—Применяется метод конечного злемента анализа конструкций для нелинейной задачи стационарной ползучести при изгибе тонких пластинок. Опредедяется формулировка конечного элемента, исходя из вариационной теоремы по аналогии к теореме для стационарной потенциальной энергии. В результате получается система совместных нелинейных алгебраических уравнений, которую решается методом Ньютона-Рафсона.

С целью получения решения задач стационарной ползучести, применяются два полно соответствующие выделенные элементы пластинки, кольцеобразный элемент шестой степены свободы и трехугольный элемент восемнадцатой степени свободы. В качестве примеров обсуждаются свободно опертые круглые пластинки, под влиянием постоянной нагрузки и нагрузки в точке и свободно опертая, постоянно нагруженная квадратная пластинка.